Vocabulary

Review
1. Circle each word whose meaning is similar to that of fundamental.
   basic  essential  extra  necessary  secondary  unimportant

Vocabulary Builder

permutation (noun) pur myoo TAY shun
Related Word: combination (noun)
Definition: A permutation is an arrangement of items in a particular order.
Main Idea: When you put a set of objects in a certain order, you make a permutation.

Use Your Vocabulary

2. Write a permutation of each set. Sample answers are given.
   \[ \{D, N, C, U\} \quad \{A, W, 6, 7, 2, E, V\} \quad \{y, o, U, R\} \]
   \[ \{D, U, N, C\} \quad \{W, A, V, E, 6, 7, 2\} \quad \{U, o, y, R\} \]

3. Write all possible permutations of the numbers 5, 9, and 8.
   \[ 5, 9, 8; 5, 8, 9; 8, 5, 9; 8, 9, 5; 9, 8, 5; 9, 5, 8 \]

4. There are 6 permutations of the numbers 5, 9, and 8.

Problem 1 Using the Fundamental Counting Principle

Got It? In 1966, one type of Maryland license plate had two letters followed by four digits. How many of this type of plate were possible?

5. Multiple Choice Which set of letters and digits gives a possible 1966 Maryland license plate?
   \[ A \rightarrow TV 432 \quad B \rightarrow RC 2301 \quad C \rightarrow KPH 621 \quad D \rightarrow OMNQ 23 \]
6. There are 26 possible letters for each letter in the license plate.

There are 10 possible digits for each digit.

7. Use the Fundamental Counting Principle to find the number of possible license plates. Complete the expression.

\[ 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \]

8. Circle the number of possible 1966 Maryland license plates.

[26,000 45,697,600 6,760,000]

Using factorial notation, you can write 3 \( \cdot \) 2 \( \cdot \) 1 as 3!, read “three factorial.” For any positive integer \( n \), \( n \) factorial is \( n! = n(n - 1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \). The zero factorial is 0! = 1.

**Problem 2** Finding the Number of Permutations of \( n \) Items

**Got It?** In how many ways can you arrange 8 shirts on hangers in a closet?

9. Complete the model below.

<table>
<thead>
<tr>
<th>Relate</th>
<th>1st shirt</th>
<th>2nd shirt</th>
<th>3rd shirt</th>
<th>4th shirt</th>
<th>5th shirt</th>
<th>6th shirt</th>
<th>7th shirt</th>
<th>8th shirt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

10. The total number of permutations is

\[ 8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40,320 \]

**Key Concept** Number of Permutations

The number of permutations of \( n \) items of a set arranged \( r \) items at a time is

\[ n^P_r = \frac{n!}{(n - r)!} \text{ for } 0 \leq r \leq n. \]

Example: \( 10^P_4 = \frac{10!}{6!} = 5040 \)

11. Why can’t \( r \) be greater than \( n \)?

Answers may vary. Sample: \( n \) is the number of items you have. \( r \) is the number of items you are selecting. You cannot select more items than you have.
Problem 3

Finding \( nPr \)

Got It? In how many ways can 15 runners finish first, second, and third?

12. Use the permutation formula. Circle the value of \( n \), the number of runners in the set. Underline the value of \( r \), the number of runners arranged at a time.

13. Use the justifications at the right to find the number of ways in which 15 runners can finish first, second, and third.

\[
nPr = \frac{n!}{(n-r)!} \quad \text{Write the formula.}
\]

\[
= \frac{15!}{(15-3)!} \quad \text{Substitute } n \text{ and } r.
\]

\[
= \frac{15!}{12!} = 2730 \quad \text{Simplify.}
\]

Key Concept

Number of Combinations

The number of combinations of \( n \) items of a set chosen \( r \) items at a time is

\[
C_r = \frac{n!}{r!(n-r)!} \quad \text{for } 0 \leq r \leq n
\]

Example:

\[
5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3! \cdot 2!} = \frac{120}{6 \cdot 2} = 10
\]

14. Which is greater, \( 5C_3 \) or \( 5P_3 \)? Explain.

\[
5P_3 > 5C_3 \text{ because } 5P_3 = \frac{n!}{(n-r)!} = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60
\]

\[
\text{and } 5C_3 = 10
\]

Problem 4

Finding \( nCr \)

Got It? What is the value of \( 8C_3 \)?

15. Cross out the equations that do NOT give the correct formula for \( 8C_3 \).

16. Simplify the remaining equation from Exercise 15.

\[
8C_3 = \frac{8!}{3!(8-3)!} = \frac{8!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3! \cdot 5!} = 56
\]
Problem 5 Identifying Whether Order Is Important

Got It? A chemistry teacher has a class work in groups to draw the molecular structure of water. Each group submits one drawing. There are eight groups. The teacher selects the four drawings that earn the highest grades. In how many ways can he select and arrange the four drawings from left to right on the wall?

17. Circle the formula you will use to solve this problem.

\[
\begin{align*}
\binom{n}{r} &= \frac{n!}{r!(n-r)!} \\
\text{or} \\
n^P_r &= \frac{n!}{(n-r)!}
\end{align*}
\]

18. Identify each value.

\[
\begin{align*}
n &= 8 \\
r &= 4
\end{align*}
\]

19. In how many different ways can the teacher select and arrange the drawings?

\[
8^P_4 = \frac{8!}{(8-4)!} = \frac{8!}{4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!} = 1680
\]

Lesson Check • Do you UNDERSTAND?

Reasoning Use the definition of permutation to show why 0! should equal 1.

20. Circle the equation that shows the Fundamental Counting Principle and the Permutation Formula for \(n\) items arranged \(n\) at a time.

\[
\begin{align*}
0! &= \frac{n!}{(n-0)!} \\
n! &= \frac{n!}{(n-n)!} \\
n! &= \frac{n!}{(n-0)!}
\end{align*}
\]

21. Simplify the equation you chose in Exercise 20.

22. Underline the correct expressions to complete the sentence.

For \(\frac{n!}{0!}\) to equal \(0! / n!\), 0! must equal \(0! / 1\).

Math Success

Check off the vocabulary words that you understand.

☐ Fundamental Counting Principle ☐ permutation ☐ \(n\) factorial ☐ combination

Rate how well you can find permutations and combinations.

Need to review 0 2 4 6 8 10 Now I get it!
Vocabulary

Review

1. Draw a line from each experiment in Column A to a corresponding set of possible outcomes in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toss a coin.</td>
<td>G, R, B</td>
</tr>
<tr>
<td>Roll a six-sided number cube.</td>
<td>heads or tails</td>
</tr>
<tr>
<td>Draw a ball at random from a box holding 1, 2, 3, 4, 5, 6 1 green, 1 red, and 1 blue ball.</td>
<td>1, 2, 3, 4, 5, 6</td>
</tr>
</tbody>
</table>

Vocabulary Builder

**theoretical probability** (noun)

**Definition:** The theoretical probability of an event is the ratio of the number of ways that the event can occur to the total number of equally likely outcomes in the sample space.

**Example:** A student rolls a six-sided number cube. The theoretical probability that the student rolls an even number is

\[ P(\text{even}) = \frac{\text{number of ways to roll an even number}}{\text{number of possible outcomes}} = \frac{3}{6} \]

Use Your Vocabulary

2. Write T for true or F for false.

**F**

*Theoretical probability* is the sum of the number of ways an event can occur and the number of possible outcomes.

**T**

The ratio of the number of ways for an event to occur to the total number of possible outcomes is the theoretical probability.
Got It? A softball player got a hit in 20 of her last 50 times at bat. What is the experimental probability that she will get a hit in her next at bat?

3. Use the words in the box at the right to complete the ratio. Then substitute and simplify.

\[
P(\text{hit}) = \frac{\text{number of hits}}{\text{number of times at bat}}
\]

\[
= \frac{20}{50} = 0.40 = 40\%
\]

Got It? What is the theoretical probability of getting a sum that is an odd number on one roll of two fair number cubes?

4. The table shows the possible sums for one roll of two number cubes. Circle the favorable outcomes.

5. There are 18 favorable outcomes.

6. Complete and simplify the ratio.

\[
P(\text{odd number}) = \frac{18}{36} = \frac{1}{2}
\]

Got It? What is the theoretical probability of being dealt all four 7’s in a 5-card hand?

7. Complete the reasoning model below.

<table>
<thead>
<tr>
<th>Think</th>
<th>Write</th>
</tr>
</thead>
<tbody>
<tr>
<td>First, I find the number of combinations of four 7’s from four 7’s.</td>
<td>[ _4 C_4 = \frac{4!}{(4 - 4)!} = 1 ]</td>
</tr>
<tr>
<td>A five-card hand with four 7’s has one non-7 card. I find the number of combinations of one non-7 from 48 remaining cards.</td>
<td>[ _{48} C_1 = \frac{48!}{(48 - 1)!} \cdot \frac{47!}{1!} = \frac{48!}{47!} = 48 ]</td>
</tr>
<tr>
<td>I multiply to find the number of 5-card hands with four 7’s.</td>
<td>[ _4 C_4 \cdot _{48} C_1 = 1 \cdot 48 = 48 ]</td>
</tr>
</tbody>
</table>
8. Find the total number of possible 5-card hands.

\[
\binom{52}{5} = \frac{52!}{5!(52-5)!} = \frac{52!}{5!(47)!} = 2,598,960
\]

9. Use your answers to Exercises 7 and 8 to write the probability.

\[
P(\text{hand with four 7's}) = \frac{\text{5-card hands with four 7's}}{\text{possible 5-card hands}}
\]

10. Substitute and simplify.

\[
P(\text{hand with four 7's}) = \frac{48}{2,598,960} = \frac{1}{54,154}
\]

---

**Problem 5** Finding Geometric Probability

**Got It?** Suppose a batter’s strike zone is 15 in. by 20 in. and his high-inside strike zone is 3 in. by 5 in. What is the probability that a baseball thrown at random within the strike zone will be a high-inside strike?

11. Find the area of the batter’s strike zone.

\[
A(\text{strike zone}) = 15 \cdot 20 = 300 \text{ in.}^2
\]

12. Find the area of the batter’s high-inside strike zone.

\[
A(\text{high-inside}) = 3 \cdot 5 = 15 \text{ in.}^2
\]

13. Complete the equation to solve the problem. Round your final answer to two decimal places.

\[
P(\text{high-inside strike}) = \frac{\text{area of high-inside strike zone}}{\text{area of total strike zone}} = \frac{15}{300} = 0.05
\]

14. The probability that a baseball thrown at random within the strike zone will be a high-inside strike is 5\%.

---

**Lesson Check** Do you know HOW?

Find \(P(3)\) when rolling a fair number cube.

15. Complete.

\[
P(3) = \frac{\text{number of ways you can roll 3}}{\text{number of possible outcomes}} = \frac{1}{6}
\]

Find \(P(2 \text{ or } 4)\) when rolling a fair number cube.


\[
P(2 \text{ or } 4) = \frac{\text{number of ways you can roll 2 or 4}}{\text{number of possible outcomes}} = \frac{2}{6} = \frac{1}{3}
\]
Lesson Check • Do you UNDERSTAND?

Reasoning  Why is a simulation better the more times you perform it?

17. Using your graphing calculator, enter randInt (1, 2, 5). This will generate a list of 5 outcomes of 1 or 2. Let 1 represent a tossed coin landing heads-up and let 2 represent a tossed coin landing tails-up. Record your results in the table.

\[
\begin{array}{cccccc}
2 & 1 & 1 & 1 & 2 \\
\end{array}
\]

**Answers may vary. Sample:**

18. Repeat the experiment two more times with 10 and 20 in place of the 5. Record the results. **Answers may vary. Sample:**

\[
\begin{array}{ccccccccccc}
2 & 1 & 2 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 \\
2 & 1 & 2 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 \\
1 & 2 & 2 & 1 & 2 & 2 & 1 & 1 & 1 & 2 & 2 \\
\end{array}
\]

19. Find the experimental probability of landing on heads for each experiment above.

\[
P(\text{heads}) = \frac{3}{5} = 0.6 \\
P(\text{heads}) = \frac{6}{10} = 0.6 \\
P(\text{heads}) = \frac{10}{20} = 0.5
\]

**Answers may vary. Check students’ work.**

20. Explain why a simulation is better the more times you perform it.

**Answers may vary. Sample:** The more times you perform a simulation, the closer the experimental probability gets to the theoretical probability.

Math Success

Check off the vocabulary words that you understand.

- [ ] experimental probability
- [ ] simulation
- [ ] theoretical probability

Rate how well you can determine the probability of events.

Need to review: 0 2 4 6 8 10  
Now I get it!
11-3 Probability of Multiple Events

Review

1. Cross out all numbers that are NOT multiples of 8.
   48       74       405       136

2. Write three numbers that are multiples of both 2 and 9.
   Answers may vary. Samples are given.
   18  72  36

Vocabulary Builder

**event** (noun) ee VENT

Math Usage: An event is one or more outcomes from the set of all possible outcomes of an experiment.

Example: Roll a number cube. The set of all possible outcomes is {1, 2, 3, 4, 5, 6}. The set of outcomes resulting in the event that an even number is rolled is the set {2, 4, 6}.

Use Your Vocabulary

Write a possible event for each experiment. Answers may vary. Sample answers are given.

3. A basket has 5 red balls and 3 blue balls. You pick one ball without looking.
   You pick a red ball.

4. You roll a number cube one time.
   An odd number comes up.

Problem 1 Classifying Events

Got It? You select a coin at random from your pocket. You replace the coin and select again. Are your selections independent events? Explain.
5. Circle the true statement.

Selecting the first coin affects the possible outcomes of picking the second coin, because you replace the coin.

6. The two selections are independent / dependent.

Key Concept  Probability of Compound Events

**Probability of A and B** If A and B are independent events, then \( P(A \text{ and } B) = P(A) \cdot P(B) \).

**Probability of A or B** If A and B are not mutually exclusive, then \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \).

If A and B are mutually exclusive, then \( P(A \text{ or } B) = P(A) + P(B) \).

Events A and B are independent and mutually exclusive. \( P(A) = \frac{3}{5} \) and \( P(B) = \frac{4}{9} \).

Write T for true or F for false.

7. \( P(A \text{ and } B) = \frac{4}{15} \)  

8. \( P(A \text{ or } B) = \frac{3}{5} + \frac{4}{9} - \frac{4}{15} \)

Problem 2  Finding the Probability of Independent Events

**Got It?** At a picnic there are 10 diet drinks and 5 regular drinks. There are also 8 bags of fat-free chips and 12 bags of regular chips. If you grab a drink and a bag of chips without looking, what is the probability that you get a regular drink and regular chips?

9. Define each event.

Event A = You pick a regular drink.  

Event B = You pick a bag of regular chips.

10. Complete each equation.

\[
P(A) = \frac{\text{number of regular drinks}}{\text{total number of drinks}} = \frac{5}{15}
\]

\[
P(B) = \frac{\text{number of bags of regular chips}}{\text{total number of bags of chips}} = \frac{12}{20}
\]

11. Use the justifications at the right to find the probability.

\[
P(A \text{ and } B) = P(A) \cdot P(B) \quad \text{Multiply to find the probability of independent events.}
\]

\[
= \frac{5}{15} \cdot \frac{12}{20} \quad \text{Substitute.}
\]

\[
= \frac{1}{5}, \text{ or } 0.2 \quad \text{Simplify.}
\]
Problem 3  Mutually Exclusive Events

Got It?  You roll a standard number cube. Are the events rolling an even number and rolling a prime number mutually exclusive? Explain.

12. The numbers on the number cube are listed below. Circle the even numbers.

| 1 | 2 | 3 | 4 | 5 | 6 |

13. Are the events mutually exclusive? Explain.

No. Explanations may vary. Sample: You can roll an even number and a prime number at the same time.

Problem 4  Finding Probability for Mutually Exclusive Events

Got It?  Languages  At your high school, a student can take one foreign language each term. About 37% of the students take Spanish. About 15% of the students take French. About 9% of the students take Mandarin Chinese. What is the probability that a student chosen at random is taking Spanish, French, or Mandarin Chinese?


Yes. Explanations may vary. Sample: The events are mutually exclusive because a student can take only one foreign language each term.

15. Circle the formula you would use to find the probability.

\[ P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) \]

16. Find the probability that a student is taking Spanish, French, or Mandarin Chinese.

\[
\begin{align*}
P(\text{Spanish or French or Chinese}) &= P(\text{Spanish}) + P(\text{French}) + P(\text{Chinese}) \\
&= 0.37 + 0.15 + 0.09 \\
&= 0.61, \text{ or about } 61%
\end{align*}
\]

Problem 5  Finding Probability

Got It?  Suppose you reach into the dish at the right and select a token at random. What is the probability that the token is square or red (R)?

17. Complete.

\[
\begin{align*}
\text{number of tokens in the dish} & = 9 \\
\text{number of square tokens} & = 3 \\
\text{number of red (R) tokens} & = 3 \\
\text{number of tokens that are square and red (R)} & = 1
\end{align*}
\]
18. Use your answers to Exercise 17 to find each probability.

\[
P(\text{square token}) = \frac{3}{9} \quad P(\text{red (R) token}) = \frac{3}{9} \quad P(\text{square and red (R) token}) = \frac{1}{9}
\]

19. Find the probability that the token you select is square or red (R).

\[
\frac{3}{9} + \frac{3}{9} - \frac{1}{9} = \frac{5}{9}
\]

**Lesson Check • Do you UNDERSTAND?**

**Error Analysis** The weather forecast for the weekend is a 30% chance of rain on Saturday and a 70% chance of rain on Sunday. Your friend says that means there is a 100% chance of rain this weekend. What error did your friend make?

20. Which formula should you use to find the chance of rain for this weekend? Circle your answer.

\[
P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad P(A \text{ and } B) = P(A) \cdot P(B)
\]

21. Explain your friend’s error.

Answers may vary. Sample: My friend added together the chance of rain on Saturday and the chance of rain on Sunday. My friend did not subtract the probability that it will rain on Saturday AND Sunday.

**Math Success**

Check off the vocabulary words that you understand.

- event
- independent events
- mutually exclusive events

Rate how well you can find probabilities of multiple events.
11-4 Conditional Probability

Vocabulary

Review

1. **Multiple Choice** A basket has 7 plain, 8 wheat, 2 raisin, and 3 blueberry bagels. What is the probability of selecting a plain bagel without looking?

   - A $\frac{3}{20}$
   - B $\frac{7}{20}$
   - C $\frac{1}{10}$
   - D $\frac{2}{5}$

2. A pencil machine contains 15 blue, 16 red, 18 green, and 26 purple pencils. The machine randomly dispenses a pencil when one is purchased. Keisha buys a pencil. Circle the probability that the pencil she buys is red.

   - $\frac{1}{16}$
   - $\frac{4}{25}$
   - $\frac{16}{75}$
   - $\frac{59}{75}$
   - 16

Vocabulary Builder

**conditional** (adjective) kun DISH un ul

Related Words: conditions (adjective), conditionally (adjective), conditioned (adjective), conditioning (noun or adjective)

Definition: A conditional statement is a sentence stating that the probability of one event depends on the occurrence of another event.

Use Your Vocabulary

Complete each statement with the correct word from the list. Use each word only once.

- conditioning
- conditions
- conditioned
- condition

3. The ___ in the classroom made concentration difficult.

4. The track and field athletes were having spring ___.

5. Matthew had a heart ___ that made participating in physical activities difficult.

6. The dog was ___ to ring the bells on the door when he wanted to go outside.
**Problem 1** Finding Conditional Probability

**Got It?** The table shows the number of students of each gender at two-year and four-year colleges and graduate schools in 2005. What is \( P(\text{four-year} \mid \text{male}) \)?

7. Find the total number of male students.
   \[
   1866 + 4324 + 1349 = 7539
   \]

8. Complete the steps to solve the problem. Round your answer to the nearest hundredth.
   \[
   P(\text{four-year} \mid \text{male}) = \frac{\text{number of males attending four-year colleges}}{\text{total number of male students}}
   \]
   \[
   = \frac{4324}{7539} = 0.57
   \]

**Got It?** Reasoning Without calculating, given a student is enrolled in a four-year college, is it more likely for the student to be male or female? Explain.

9. There are 4324 male students enrolled in four-year colleges.

10. There are 5517 female students enrolled in four-year colleges.

11. Is it more likely for a four-year college student to be male or female? Explain.

   Female. Explanations may vary. Sample: There are more female four-year college students than male four-year college students.

**Problem 2** Conditional Probability in Statistics

**Got It?** Americans recycle increasing amounts through municipal waste collection. The table shows the collection data for 2007. What is the probability that a sample of recycled waste is plastic?

12. Find the total number of tons (in millions) of recycled waste.
   \[
   45.2 + 7.2 + 3.2 + 2.1 + 21.7 = 79.4
   \]

13. Circle the amount of recycled waste that is plastic.

   \[
   \text{Plastic} = 2.1
   \]

   \[
   P(\text{plastic} \mid \text{recycled}) = \frac{\text{plastic}}{\text{total waste recycled}} = \frac{2.1}{79.4} \approx 0.026
   \]
15. The probability that a sample of recycled waste is plastic is about \( 2.6 \) %.

### Key Concept
**Conditional Probability**

For any two events \( A \) and \( B \), with \( P(A) \neq 0 \),

\[
P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}.
\]

### Problem 3 Using the Conditional Probability Formula

**Got It? Market Research** Researchers asked shampoo users whether they apply shampoo directly to the head, or indirectly using a hand. What is the probability that a respondent applies shampoo directly to the head, given that the respondent is female?

16. Determine each probability.

\[
P(\text{female}) = \frac{30}{50} \quad P(\text{female and directly to head}) = \frac{6}{50}
\]

17. Circle the form the conditional probability formula will have.

\[
P(\text{directly } \mid \text{ female}) = \frac{P(\text{female})}{P(\text{female and directly to head})}
\]

18. Use your answers to Exercises 16 and 17 to find the probability that a female respondent applies shampoo directly onto her head.

\[
P(\text{directly } \mid \text{ female}) = \frac{P(\text{female and directly to head})}{P(\text{female})} = \frac{6/50}{30/50} = \frac{6}{30} = \frac{1}{5} = 0.2
\]

19. The probability that a female respondent applies shampoo directly onto her head is \( 0.2 \).

### Problem 4 Using a Tree Diagram

**Got It? Education** A school system compiled the following information from a survey it sent to people who were juniors 10 years earlier.

- 85% of the students graduated from high school.
- Of the students who graduated from high school, 90% are happy with their present jobs.
- Of the students who did not graduate from high school, 60% are happy with their present jobs.

What is the probability that a student from the junior class 10 years ago did not graduate and is happy with his or her present job?
20. In the tree diagram at the right, \( G \) = graduated, \( \text{NG} \) = not graduated, \( H \) = happy with present job, and \( \text{NH} \) = not happy with present job. Use the numbers in the shaded box to complete the tree diagram below. Use each number once.

21. Highlight the path on the tree diagram that shows the probability that a person who did not graduate is happy with his or her present job.

22. Calculate the probability.
\[
P(\text{NG and } H) = P(\text{NG}) \cdot P(H | \text{NG}) = 0.15 \cdot 0.60 = 0.09
\]

Lesson Check • Do you know HOW?

The probability that a car has two doors, given that it is red, is 0.6. The probability that a car has two doors and is red is 0.2. What is the probability that a car is red?

23. Circle the equation you will use to solve this problem.
\[
\frac{0.6}{P(\text{red})} = \frac{0.2}{P(\text{red})} = P(\text{red}) = 0.2 \cdot 0.6
\]

24. Solve the equation you circled in Exercise 23.
\[
P = \frac{0.2}{0.6} = 0.33
\]

25. The probability that a car is red is \(33\frac{1}{3}\)%. 

Math Success

Check off the vocabulary words that you understand.

conditional probability

Rate how well you can determine conditional probability.

Need to review 0 2 4 6 8 10 Now I get it!
Vocabulary

Review

1. Circle the type of probability that is equal to the ratio of the number of times an event occurs divided by the number of trials.

   theoretical probability  experimental probability  geometric probability

2. Fill in the blanks to complete the sentence.

   The possible values of probabilities are real numbers between 0 and 1, inclusive.

Vocabulary Builder

Probability model  (noun)  prä·bä·Bl-a·tē

Related Words: outcome, fair decision, simulation, equally likely

Definition: A probability model is a model used to assign probabilities to outcomes of a chance process. A simulation is an example of a probability model.

Example: One student will be selected at random from four volunteers for class representative. You can use a spinner divided into four equal sections to predict the chances of each student being selected.

Use Your Vocabulary

3. Fill in the blanks to complete the sentence.

   If a sample space has \( n \) equally likely outcomes and an event \( A \) occurs in \( m \) of these outcomes, then the theoretical probability of event \( A \) is

   \[ P(A) = \frac{m}{n} \]

4. Circle the spinner that has equally likely outcomes.
Problem 1  Making a Fair Decision

Got It? Two siblings are trying to decide who has to mow the lawn this weekend. They decide to race, and the winner does not have to mow the lawn. Is the result a fair decision?

5. Circle all the things you do not know about the siblings.

<table>
<thead>
<tr>
<th>age</th>
<th>gender</th>
<th>weight</th>
</tr>
</thead>
</table>

6. Underline the correct word to complete the sentence.

It is very likely / unlikely the two siblings have the same speed when running a race.

7. Do the siblings have an equal chance of winning the race? Explain.

No; Answers may vary. Sample: One sibling is probably faster than the other. It is very unlikely that they both run at the same speed.

8. Underline the correct word to complete the sentence.

The result is a(n) fair / unfair decision.

Problem 2  Using Random Numbers

Got It? A teacher wants to organize 10 students into two teams for a math game. The teacher assigns each student a number between 0 and 9. He uses the second line of digits from the random number table below to select the teams. He alternates the assigned team as each student is chosen. Which numbers will be used to create team 1?

<table>
<thead>
<tr>
<th>Random Number Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>87494 39707 20525 95704 48361 27556 34599</td>
</tr>
<tr>
<td>14164 15888 24997 82392 08525 47551 37304</td>
</tr>
<tr>
<td>61249 08241 16243 18371 03349 91759 53613</td>
</tr>
<tr>
<td>67868 56747 73521 05975 40411 49493 70904</td>
</tr>
</tbody>
</table>

9. Circle the second row of digits in the table. Rewrite the numbers in the second row as a list of single digits. Starting with the first number on the left, circle each digit from 0 to 1 the first time it is shown. Put an X through any duplicates.

144 x 6 5 7 8 9 0 x 3 0 x 1 7 0 9 0 4

10. Alternate the circled numbers between team 1 and team 2 to fill in the blanks.

<table>
<thead>
<tr>
<th>Numbers for Team 1</th>
<th>Number for Team 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 6 8 9 3</td>
<td>4 5 2 7 0</td>
</tr>
</tbody>
</table>
Problem 3  Modeling with a Simulation

Got It?  Suppose that you are playing a board game for which you must roll a 6 on a number cube before you are able to move your game piece from start. Describe a simulation you can use to predict the number of times you would expect to have to roll the number cube before you can move from start.

Know
You know you must roll a 6 on a number cube before you can move your game piece from start.

Need
You need to find a probability model that generates equally-likely events.

Plan
You can use a spinner with 6 equal sections to simulate the number of times you have to roll the number cube before you can move from start.

11.  Step 1  The results of 5 trials are shown in the table. Complete the table to show the number of spins until the spinner lands on 6 for each trial.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Individual Spin Results</th>
<th>Spins until 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5, 1, 6</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5, 6</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3, 3, 6</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>3, 3, 1, 5, 3, 2, 6</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>1, 6</td>
<td>2</td>
</tr>
</tbody>
</table>

12.  Step 2  The number of spins until 6 occurs for 20 more trials are shown below.
3, 8, 2, 11, 4, 4, 2, 6, 4, 7, 8, 2, 11, 3, 3, 1, 5, 1, 4, 12

13.  Step 3  Use the results from Steps 1 and 2 to find the average number of rolls until a 6.

\[
\frac{120}{25} = 4.8
\]

14.  On the average, you will need to roll 4.8 times until you can start the game.

Problem 4  Using Probability to Analyze Decisions

Got It?  A pharmaceutical company is testing the effectiveness of a new drug for asthma patients. The results of a test are shown in the contingency table below. Should the company produce and distribute the new drug?

<table>
<thead>
<tr>
<th></th>
<th>Reported improvements</th>
<th>Did not report improvements</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Received the drug</td>
<td>23</td>
<td>27</td>
<td>50</td>
</tr>
<tr>
<td>Received the placebo</td>
<td>19</td>
<td>31</td>
<td>50</td>
</tr>
<tr>
<td>Totals</td>
<td>42</td>
<td>58</td>
<td>100</td>
</tr>
</tbody>
</table>
15. What is the probability a volunteer reported noticeable improvement in symptoms given the volunteer received the test drug?

\[ P(\text{improvement} \mid \text{drug}) = \frac{\text{number of volunteers who improved}}{\text{number of volunteers who received the drug}} = \frac{23}{50} = 0.46 \]

16. What is the probability a volunteer received the placebo given the volunteer did not report a noticeable improvement in symptoms?

\[ P(\text{placebo} \mid \text{no improvement}) = \frac{31}{58} \approx 0.5345 \]

17. The results are around 50-50. The company should/should not produce this new drug.

**Lesson Check • Do you UNDERSTAND?**

A classmate conducted a simulation to predict how many boxes of cereal he would need to buy to get all 5 prizes. After one trial of the simulation, he concluded that he would need to buy 7 boxes of cereal to get all 5 prizes. Do you agree with your classmate’s conclusion? Explain.

18. Circle the correct answer for each question. In a simulation to predict how many boxes of cereal you would need to buy to get all 5 prizes, what is the least possible number of boxes you would need to buy?

4   5   6   7   10   15   undetermined

What is the greatest possible number of boxes you would need to buy?

4   5   6   7   10   15   **undetermined**

19. After one trial, you get all 5 prizes when you buy 7 boxes of cereal. Does that mean that you will get all 5 prizes with the next 7 boxes you buy? Explain. **Answers may vary.**

**Sample:** No. It is possible to get all 5 prizes buying only 5 or 6 boxes. It is also possible that it will take more than 7 boxes to get all the prizes.

20. Do you agree with your classmate’s conclusion? Explain. **Answers may vary.**

**Sample:** No. A simulation of one trial is not a good probability model.

**Math Success**

Check off the vocabulary words that you understand.

- probability model
- fair decision
- simulation
- two-way frequency table

Rate how well you can use probability models to assign probabilities of a chance process.
**Review**

1. Find the *median* of the set of numbers.

   1, 5, 7, 9, 3, 12, 7, 6

   Order the data: 1, 3, 5, 6, 7, 7, 9, 12.
   The middle two values are 6 and 7.
   The median is \( \frac{6 + 7}{2} = 6.5 \).

**Vocabulary Builder**

**quartile** *(noun)* \( \text{KWAWR tyyl} \)

Related Words: interquartile range, median

**Definition:** A *quartile* is one of three numbers, \( Q_1, Q_2, Q_3 \), that divide an ordered data set into four parts. Each part includes the same number of data values.

**Example:** The *quartiles* of the data set 1, 2, 3, 6, 6, 8, 10 are \( Q_1 = 2, Q_2 = 6, \) and \( Q_3 = 8 \). The second *quartile*, \( Q_2 = 6 \), is also the median of the data set.

**Use Your Vocabulary**

2. Write **T** for *true* or **F** for *false*.

   - **T** You must order a data set before finding the *quartiles*.
   - **F** Every data set of numbers has four *quartiles*.
   - **F** The number \( Q_2 \) represents the first *quartile*.
   - **F** *Quartiles* divide an ordered data set into three parts.
### Key Concepts  Measures of Central Tendency

<table>
<thead>
<tr>
<th>Measure</th>
<th>Definition</th>
<th>Example, using 1, 2, 3, 3, 4, 5, 5, 9</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>sum of the data values number of data values</td>
<td>[ \frac{1 + 2 + 3 + 3 + 4 + 5 + 5 + 9}{8} = 4 ]</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>For a data set listed in order: The middle values for an odd number of data values; the mean of the two middle values for an even number of data values</td>
<td>For 1, 2, 3, 3, 4, 5, 5, 9 the middle values are 3 and 4. The median is the mean ( \frac{3 + 4}{2} = 3.5 ).</td>
</tr>
<tr>
<td><strong>Mode</strong></td>
<td>the most frequently occurring value(s)</td>
<td>Two modes: In 1, 2, 3, 3, 4, 5, 5, 9 both 3 and 5 occur twice.</td>
</tr>
</tbody>
</table>

3. Since the data set 18, 21, 25, 16, 14, 29, 35 has an **even** number of values, the median is **16 / 21 / 23**.

### Problem 2: Identifying an Outlier

**Got It?** Suppose the values 56, 65, 73, 59, 98, 65, 59 are the number of customers in a small restaurant each night in one week. Would you discard the outlier? Explain.

4. Next to each reason below, write D if it is a reason for **discarding** the outlier. Write K if it is reason for **keeping** the outlier.

- 98 is very different from the other data values and will skew the mean. **D**
- 98 could represent the number of customers on a Friday or Saturday night. **K**
- The numbers could be used to determine how much food to order for the restaurant. **K**
- 98 might represent an inaccurate count since it is so different from the other data values. **D**
- The numbers could be used to decide whether there is a consistent need for a larger restaurant. **D**

### Problem 3: Comparing Data Sets

**Got It?** Temperature  The table shows average monthly water temperatures for two locations on the Gulf of Mexico. How can you compare the 12 water temperatures from Dauphin Island with the 12 water temperatures from Grand Isle?

<table>
<thead>
<tr>
<th>Gulf of Mexico Eastern Coast Water Temperatures (°F)</th>
<th>Location</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dauphin Island, Alabama</td>
<td>51 61</td>
<td>53 64</td>
<td>60 70</td>
<td>70 75</td>
<td>82 83</td>
<td>84 85</td>
<td>84 86</td>
<td>80 85</td>
<td>72 77</td>
<td>62 70</td>
<td>56 65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grand Isle, Louisiana</td>
<td>61 61</td>
<td>61 64</td>
<td>70 77</td>
<td>70 77</td>
<td>83 83</td>
<td>85 85</td>
<td>85 83</td>
<td>83 83</td>
<td>77 77</td>
<td>70 70</td>
<td>65 65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Source:** National Oceanographic Data Canter
5. Find the sum of temperatures for each location.

\[
\begin{align*}
\text{Dauphin Island} & \quad 51 + 53 + 60 + 70 + 75 + 82 + 84 + 84 + 80 + 72 + 62 + 56 \\
& = 829 \\
\text{Grand Isle} & \quad 61 + 61 + 64 + 70 + 77 + 83 + 85 + 85 + 83 + 77 + 70 + 65 \\
& = 881
\end{align*}
\]

6. Find the mean temperature for each location.

\[
\begin{align*}
\text{Dauphin Island} & \quad \frac{829}{12} = 69.1 \\
\text{Grand Isle} & \quad \frac{881}{12} = 73.4
\end{align*}
\]

7. Find the mode(s), minimum value, maximum value, and range.

<table>
<thead>
<tr>
<th>Location</th>
<th>Mode(s)</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dauphin Island</td>
<td>84</td>
<td>51</td>
<td>84</td>
<td>84 - 51 = 33</td>
</tr>
<tr>
<td>Grand Isle</td>
<td>61, 70, 77, 83, 85</td>
<td>61</td>
<td>85</td>
<td>85 - 61 = 24</td>
</tr>
</tbody>
</table>

8. Use the ordered temperatures below. Circle the middle two temperatures for each location. Draw a box around the middle two temperatures in the lower half of the data set. Underline the middle two temperatures in the upper half of the data set.

\[
\begin{align*}
\text{Dauphin Island} & \quad 51, 53, 56, 60, 62, 70, 72, 75, 80, 82, 84, 84 \\
\text{Grand Isle} & \quad 61, 61, 64, 65, 70, 70, 77, 83, 83, 85, 85
\end{align*}
\]

9. Use your answers to Exercise 8 to complete the table.

<table>
<thead>
<tr>
<th>Location</th>
<th>Median</th>
<th>Quartile 1</th>
<th>Quartile 3</th>
<th>Interquartile Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dauphin Island</td>
<td>(\frac{70 + 72}{2} = 71)</td>
<td>(\frac{56 + 60}{2} = 58)</td>
<td>(\frac{80 + 82}{2} = 81)</td>
<td>(Q_3 - Q_1 = 81 - 58 = 23)</td>
</tr>
<tr>
<td>Grand Isle</td>
<td>(\frac{70 + 77}{2} = 73.5)</td>
<td>(\frac{64 + 65}{2} = 64.5)</td>
<td>(\frac{83 + 83}{2} = 83)</td>
<td>(Q_3 - Q_1 = 83 - 64.5 = 18.5)</td>
</tr>
</tbody>
</table>

A percentile is a number from 0 to 100 that you can associate with a value \(x\) from a data set. If \(x\) is at the 63rd percentile, then 63% of the data are less than or equal to \(x\).

10. If 71% of the data in a set are less than or equal to 269, then 269 is at the 70th / 71st / 72nd percentile of the data set.
**Problem 5** Finding Percentiles

**Got It? Testing** Here is an ordered list of midterm test scores for a Spanish class. What value is at the 55th percentile?

11. Multiply to find how many values fall at or below the 55th percentile.

\[
\text{Relate: } \frac{\text{number of test scores}}{\text{times percentile}} = \frac{\text{number of values at or below indicated percentile}}{}
\]

Write:

\[
\frac{20}{0.55} = 11
\]

12. 11 values fall at or below 79, the value at the 55th percentile.

**Lesson Check • Do you UNDERSTAND?**

**Error Analysis** A student found the median of the data set below. Explain the student’s error. What is the median?

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>6</td>
</tr>
<tr>
<td>85</td>
<td>4</td>
</tr>
<tr>
<td>90</td>
<td>10</td>
</tr>
<tr>
<td>95</td>
<td>1</td>
</tr>
</tbody>
</table>

13. There are 21 data values in the data set.

14. What error did the student make? **Answers may vary. Sample:**

The student found the median of 85 and 90 as if there were only one of each data value. He or she did not take into account the frequency of each value.

15. Find the median.

ordered data 80, 80, 80, 80, 80, 80, 85, 85, 85, 85, 90, 90, 90, 90, 90, 90, 90, 90, 90, 90, 95

**Math Success**

Check off the vocabulary words that you understand.

- mean  
- median  
- mode

Rate how well you can find the mean, median and mode.
11-7 Standard Deviation

Vocabulary

**Review**

1. A softball pitcher threw softballs at speeds of 55 mph, 42 mph, 62 mph, and 52 mph. Explain how you can find the mean speed of the softballs thrown.

   **Answers may vary. Sample:** Add 55 + 42 + 62 + 52 and divide the sum by 4, the number of pitches.

**Vocabulary Builder**

variance (noun) VEH-ree-uhms

**Related Words:** vary (verb), variable (noun), various (adjective)

**Definition:** Variance is a difference between what is expected and what actually occurs.

**Math Usage:** The variance is a measure of spread, calculated as the average of the square of the difference between each number and the mean of a data set.

**Use Your Vocabulary**

Complete each sentence with the correct form of the word *variance*.

<table>
<thead>
<tr>
<th>variant</th>
<th>variable</th>
<th>vary</th>
<th>various</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Use a ___ such as <em>x</em> to stand for an unknown amount in an equation.</td>
<td>variable</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. There are ___ ways to answer that question.</td>
<td>various</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. The exact wording of students’ answers may ___, but all the answers should cover the key ideas.</td>
<td>vary</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. By comparing the ___ of two data sets, you can see how spread out the data is around the mean.</td>
<td>variance</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Key Concepts  Variance and Standard Deviation**

- Find the mean, $\bar{x}$, of the $n$ values in the data set.
- Find the difference, $x_i - \bar{x}$, between each value $x_i$ and the mean.
- Square each difference, $(x_i - \bar{x})^2$.
- Find the average (mean) of these squares. This is the variance.

$$\sigma^2 = \frac{\sum (x - \bar{x})^2}{n}$$

- Take the square root of the variance. This is the standard deviation.

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

---

**Problem 1 Finding Variance and Standard Deviation**

**Got It?** What are the mean, variance, and standard deviation of these values?

52 63 65 77 80 82

6. Find the mean of the values. Round your answer to the nearest whole number.

$$\bar{x} = \frac{52 + 63 + 65 + 77 + 80 + 82}{6} = 69.8 \approx 70$$

7. Complete the table at the right.

8. Find the sum of the squares of the differences.

$$324 + 49 + 25 + 49 + 100 + 144 = 691$$

9. Use the formula for variance.

$$\sigma^2 = \frac{\sum (x - \bar{x})^2}{n} = \frac{691}{6} = 115.2$$

10. Use the formula for standard deviation.

$$\sigma = \sqrt{\sigma^2} = \sqrt{115.2} = 10.7$$
Chapter 11

Problem 2 Using a Calculator to Find Standard Deviation

Got It? Meteorology The table below displays the number of hurricanes in the Atlantic Ocean from 1992 to 2006. What are the mean and standard deviation?

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>11</td>
<td>10</td>
<td>3</td>
<td>10</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>4</td>
<td>7</td>
<td>9</td>
<td>14</td>
<td>5</td>
</tr>
</tbody>
</table>

Source: National Hurricane Center

11. Enter the data from the table into your graphing calculator. In the STAT CALC menu, select 1-VAR STAT. Circle the graphing calculator screen that shows the statistics.

1-Var Stats
\[ x = 7.266666667 \]
\[ \Sigma x = 109 \]
\[ \Sigma x^2 = 947 \]
\[ Sx = 3.2666666667 \]
\[ \sigma x = 3.213858878 \]
\[ n = 15 \]

1-Var Stats
\[ x = 8 \]
\[ \Sigma x = 120 \]
\[ \Sigma x^2 = 1240 \]
\[ Sx = 4.472135955 \]
\[ \sigma x = 4.320493799 \]
\[ n = 15 \]

1-Var Stats
\[ x = 15 \]
\[ \Sigma x = 225 \]
\[ \Sigma x^2 = 3375 \]
\[ Sx = 0 \]
\[ \sigma x = 0 \]
\[ n = 15 \]

12. The mean is approximately \( 7.3 \) and the standard deviation is approximately \( 3.2 \).

Problem 3 Using Standard Deviation to Describe Data

Got It? Meteorology Use the Atlantic Ocean hurricane data from Problem 2. Within how many standard deviations of the mean do all of the values fall?

13. Draw an \( \times \) for each data value from Problem 2 on the number line. The data value for year 12 has been placed for you.

14. Use the number line in Exercise 13. The mean is drawn for you. Determine the standard deviations above and below the mean and mark them on the number line.

\[
\begin{align*}
\text{1 standard deviation above} & \quad \text{mean} \ + \ \text{standard deviation} = 7.3 \ + \ 3.2 = 10.5 \\
\text{2 standard deviations above} & \quad \text{mean} \ + \ 2 \ \cdot \ \text{standard deviation} = 7.3 \ + \ 2 \ \cdot \ 3.2 = 13.7 \\
\text{1 standard deviation below} & \quad \text{mean} \ - \ \text{standard deviation} = 7.3 \ - \ 3.2 = 4.1 \\
\text{2 standard deviations below} & \quad \text{mean} \ - \ 2 \ \cdot \ \text{standard deviation} = 7.3 \ - \ 2 \ \cdot \ 3.2 = 0.9
\end{align*}
\]

15. Underline the correct numbers to complete the sentence.

All of the data values fall within \( 1 / 2 / 3 \) standard deviations of the mean.

Chapter 11
Lesson Check • Do you UNDERSTAND?

Compare and Contrast Three data sets each have a mean of 70. Set A has a standard deviation of 10. Set B has a standard deviation of 5. Set C has a standard deviation of 20. Compare and contrast these three sets.

16. The mean is plotted on each number line below. For each data set, draw lines to show two standard deviations above and below the mean.

![Number lines for Set A, Set B, and Set C with mean marked and two standard deviations shown]

17. Write 1, 2, or 3 to put Set A, Set B, and Set C in order from most spread out to least spread out.

2 Set A
3 Set B
1 Set C

18. How are the three data sets the same?

Answers may vary. Sample: The three data sets have the same mean.

19. How are the three data sets different?

Answers may vary. Sample: The three data sets have different standard deviations. The data values in Set C are the most spread out around the mean. The data values in Set B are closest to the mean.

Math Success

Check off the vocabulary words that you understand.

☐ measure of variation  ☐ variance  ☐ standard deviation

Rate how well you can find the standard deviation and variance of a set of data.

Need to review
0 2 4 6 8 10
Now I get it!
**Vocabulary**

**Review**

1. Circle the definition of the word *survey*.

   - to examine or inspect
   - to rely or lean on

**Vocabulary Builder**

**bias** *(noun)*

**Related Words:** biased (adjective), unbiased (adjective)

**Definition:** Bias is an influence on someone or something in an unfair way.

**Main Idea:** Bias means supporting one side of an issue or a situation.

**Math Usage:** A bias is a systematic error introduced by the sampling method of a survey.

**Use Your Vocabulary**

Complete each sentence with the correct form of the word *bias*.

<table>
<thead>
<tr>
<th>bias</th>
<th>unbiased</th>
<th>biased</th>
</tr>
</thead>
</table>

2. The news report about the mayor tried to be fair and **unbiased**.

3. Survey questions should be free from **bias**.

4. A judge should not be **biased** when presiding over court cases.

5. A newspaper article had the headline “Apex Company’s Prices Unfair.” Do you expect this article to be **biased or unbiased**? Explain.

**Biased.** Explanations may vary. Sample: The article’s headline states the opinion that the company’s prices are unfair.
Lesson 11-8

Analyzing Sampling Methods

Got It? To survey the eating habits of the community, employees of a local television station interview people visiting a food court in the mall. What sampling method are they using? Does the sample have bias? Explain.

6. Circle the sampling method used.

convenience random self-selected systematic

7. Does the sampling method have a bias? Explain. Answers may vary.

Sample: Yes, the employees are interviewing people who probably have similar eating habits since they are at a food court.

Got It? Reasoning A poll of every person in a population is a census. What is a situation that requires a census instead of a sample?

8. Cross out the situation that does NOT require a census.

You need to know the sizes of members of the band to order uniforms. You need to know whether students of a school prefer chicken or hamburger to plan school meals.

9. Open-Ended Write your own example of a situation that requires a census instead of a sample.

Answers may vary. Sample: You need to know which students of a class are going on a field trip to plan for transportation.

Key Concepts Sampling Types and Methods

Convenience sample Choose any members of the population who are conveniently and readily available.

Random sample All members of the population are equally likely to be chosen.

Self-selected sample Choose any members of the population who volunteer for the sample.

Systematic sample Order the population in some way, then select from it at regular intervals.

Problem 1 Analyzing Sampling Methods

In an observational study, you measure or observe members of a sample in such a way that they are not affected by the study.

In a controlled experiment, you divide the sample into two groups. You impose a treatment on one group but not on the other “control” group. Then you compare the effect on the treated group to the control group.

In a survey, you ask every member of the sample a set of questions.
A poorly designed study can result in unreliable statistics. An observational study does not influence behavior, but it is difficult to avoid experimenter bias in the collection of the data. An experiment allows for control over the factors that may impact results, but may be difficult and costly to conduct.

**Problem 2 Analyzing Study Methods**

**Got It?** A pharmaceuticals company asks for volunteers to test a new drug to treat high blood pressure. Half of the volunteers will be given the drug, and half will be given a placebo. The researcher will monitor the blood pressure of each volunteer. Which type of study method is the researcher using? Should the sample statistics be used to make a general conclusion about the effectiveness of the drug in the larger population?

12. For each situation, circle whether the group being observed is affected by the study or not affected by the study. Then tell whether it is an observational study (O) or a controlled experiment (E).

- **E** A student randomly sorts 20 volunteers for a study into two groups. Over 6 weeks, one group runs on a treadmill 30 minutes each school day and the other group does not run. Each volunteer's weight is recorded each school day. **Affected**

- **O** A laboratory technician records the diameter of 3 different bacterial colonies every 4 hours for 3 days. **Not Affected**

- **O** A field biologist records the type and number of each bird he observes from one location, every Friday for 12 months. **Not Affected**

- **E** A pharmaceuticals company asks for volunteers to test a new drug to treat high blood pressure. Half of the volunteers will be given the drug, and half will be given a placebo. The researcher will monitor the blood pressure of each volunteer. **Affected**

13. For the last scenario in Example 12, circle the factors that need to be considered to be able to make a general conclusion about the effectiveness of the drug.

- age
- gender
- overall health
- number of study participants

14. Suppose a study for a pharmaceutical company includes 500 participants of a variety of ages, 350 of which are female. After accounting for overall health, they find that the drug has a significant effect over 50% of the time. Should the sample statistics be used to make a general conclusion about the effectiveness of the drug in the larger population?

**Answers may vary. Sample:** No. Because even though the factors of the volunteers are random, like age and overall health, the number of females in the population in general is only 50.7% and this is not reflected in the sample.
Got It? What sampling method could you use to find the percent of residents in your neighborhood who recognize the governor of your state by name? What is an example of a survey question that is likely to yield information that has no bias?

14. What kind of sampling will give the least bias? Explain.

Answers may vary. Sample: Systematic sampling will give the least bias because you choose people from a population at regular intervals.

15. Open-Ended Write an example of a survey question that is likely to have no bias.

Answers may vary. Sample: Who is the person in this photograph?

Lesson Check • Do you UNDERSTAND?

Reasoning Would a large or small sample tend to give a better estimate of how the total population feels about a topic? Explain.

16. You want to know whether people like or dislike the new park in town. Which sample, 2 people or 200 people from the town, would give you a better idea of how the entire town feels about the new park?

Answers may vary. Sample: the sample of 200

17. Underline the correct words to complete the sentence.

A large / small sample tends to give a better estimate of how a total population feels about a topic because that way the population is better represented / overrepresented / underrepresented.

Math Success

Check off the vocabulary words that you understand.

☐ controlled experiment ☐ study ☐ survey ☐ bias

Rate how well you can analyze survey questions for bias.

Need to review 0 2 4 6 8 10 Now I get it!
**Vocabulary**

**Review**

1. Circle the *binomial* expressions.

\[
\begin{array}{c}
6a \\
7x^2 + 3x \\
8x + 4y + 3 \\
2y + 1
\end{array}
\]

2. Write the *binomials* that are factors of \(x^2 + x - 12\).

\[(x + 4)(x - 3)\]

**Vocabulary Builder**

**outcome** *(noun) ouk\text{\textipa{m}}*

*Definition:* An *outcome* is the final product or the end result.

*Example:* If you toss a coin, the two *outcomes* of the event are tossing heads or tails.

**Use Your Vocabulary**

List the *outcomes* of each event.

3. rolling a six-sided number cube

\[1, 2, 3, 4, 5, \text{ or } 6\]

4. spinning a spinner divided into four equal parts, each of which is a different color—red, blue, green, or purple

red, blue, green, or purple

5. selecting a ball from a basket (balls are red, blue, green, orange, yellow, or black)

red, blue, green, orange, yellow, or black
**Key Concept  Binomial Probability**

Suppose you have $n$ repeated independent trials, each with a probability of success $p$ and a probability of failure $q$ (with $p + q = 1$). Then, the **binomial probability** of $x$ successes in $n$ trials can be found by the following formula.

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

6. Why must $p + q = 1$?

**Answers may vary. Sample:** The sum of the probabilities of all the possible outcomes of an experiment must always be 1 or 100%. In a binomial experiment there are only two possible outcomes, so the probability of success plus the probability of failure must equal 1.

---

**Problem 1  Using a Formula to Find Probabilities**

**Got It? Merchandising** As part of a promotion, a store is giving away scratch-off cards. Each card has a 40% chance of awarding a prize. Suppose you had five cards. What is the probability that the number of cards that reveal a prize is 0? 1? 2? 3? 5?

7. Use the information from the problem and the formula $P(x) = \binom{n}{x} p^x q^{n-x}$.
   Identify the value of each variable.
   
   $n = 5$  
   $p = 0.4$  
   $q = 0.6$

8. If no card is a winner, the value of $x$ is $0$.

9. Determine the probability $P(0)$.
   
   $$P(0) = \binom{5}{0} (0.4)^0 (0.6)^5$$
   
   $$P(0) = 1 \cdot 1 \cdot 0.07776$$
   
   $$P(0) = 0.07776 \approx 0.08$$

10. Determine each probability.
   1 card 
   
   $$P(x) = \binom{5}{1} (0.4)^1 (0.6)^4$$
   
   $$P(x) = 5 \cdot (0.4) \cdot (0.6)^4$$
   
   $$P(x) \approx 0.26$$

   2 cards 
   
   $$P(x) = \binom{5}{2} (0.4)^2 (0.6)^3$$
   
   $$P(x) = 10 \cdot (0.4)^2 \cdot (0.6)^3$$
   
   $$P(x) \approx 0.35$$

   3 cards 
   
   $$P(x) = \binom{5}{3} (0.4)^3 (0.6)^2$$
   
   $$P(x) = 10 \cdot (0.4)^3 \cdot (0.6)^2$$
   
   $$P(x) \approx 0.23$$

   5 cards 
   
   $$P(x) = \binom{5}{5} (0.4)^5 (0.6)^0$$
   
   $$P(x) = 1 \cdot (0.4)^5 \cdot 1$$
   
   $$P(x) \approx 0.08$$
11. The probability that no card reveals a prize is \( \frac{8}{100} \)%.
The probability that exactly 1 card reveals a prize is \( \frac{26}{100} \)%.
The probability that exactly 2 cards reveal a prize is \( \frac{35}{100} \)%.
The probability that exactly 3 cards reveal a prize is \( \frac{23}{100} \)%.
The probability that exactly 5 cards reveal a prize is \( \frac{8}{100} \)%.

**Key Concept Binomial Theorem**

For every positive integer \( n \),
\[
(a + b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n
\]

**Problem 2 Expanding Binomials**

**Got It?** What is the binomial expansion of \( (3x + y)^4 \)?

12. Use the Binomial Theorem. Identify each value.
   \[
a = 3x \quad \quad b = y \quad \quad n = 4
\]

13. Expand the binomial.
   \[
   (3x + y)^4 = 4C0(3x)^4 + 4C1(3x)^3y + 4C2(3x)^2y^2 + 4C3(3x)y^3 + 4C4y^4
   \]
   \[
   = 1 \cdot (3x)^4 + 4 \cdot (3x)^3y + 6 \cdot (3x)^2y^2 + 4 \cdot (3x)y^3 + 1 \cdot y^4
   \]
   \[
   = 81x^4 + 108x^3y + 54x^2y^2 + 12xy^3 + y^4
   \]

**Problem 3 Applying Binomial Probability**

**Got It?** A multiple choice quiz has five questions. Each question has four answer choices. If you guess every answer, what is the probability of getting at least three correct?

14. Each question has four answer choices. The probability of guessing an answer correctly is \( p = \frac{1}{4} \).

15. The probability of guessing an answer incorrectly is \( q = \frac{3}{4} \).

16. Circle the value of \( n \), the number of questions on the test.
   
   1  2  3  4  5

17. Circle the statement that corresponds to the phrase “getting at least three correct.”
   
   answering 4 or 5 correctly
   answering 3, 4, or 5 correctly
   answering only 3 correctly
18. Evaluate the binomial probabilities.

\[
P(\text{at least 3 correct}) = \binom{5}{3}(0.75)^3(0.25)^2 + \binom{5}{4}(0.75)^4(0.25) + (0.25)^5
\]

\[
= 0.087890625 + 0.0146484 + 0.0009766
\]

\[
= 0.1035
\]

19. The probability of getting at least three questions correct by guessing is about 10.4\%.

---

**Lesson Check • Do you UNDERSTAND?**

**Vocabulary** Explain how flipping a coin 10 times meets all of the conditions for a binomial experiment.

20. Circle the two true statements about 10 coin tosses.

- Each toss has two possible outcomes.
- Every toss has equal probability of success.
- In 10 tosses, 5 outcomes will be heads and 5 will be tails.
- The probability of the outcome tails increases as the number of tosses increases.

21. Underline the correct words to complete the sentence.

In a binomial experiment, each trial must have \( \frac{1}{2} / n \) possible outcomes, and the probability of a success must be constant / varying throughout the trials.

---

**Math Success**

Check off the vocabulary words that you understand.

- binomial experiment
- binomial probability
- Binomial Theorem

Rate how well you can find binomial probabilities.

Need to review 0 2 4 6 8 10 Now I get it!
Review

1. Cross out the phrase that is NOT the definition of distribute.
   - to collect or gather
   - to give or hand out

Vocabulary Builder

**discrete** (noun) *dih* SKREEET

Related Word: continuous (adjective)

Definition: Something that is discrete is separate or distinct from something else.

Example: Roll a number cube once. There are six discrete outcomes for this experiment: 1, 2, 3, 4, 5, 6.

Non-example: A person’s body temperature is not discrete. It can take on any value in a certain interval.

Use Your Vocabulary

Write D if the experiment has discrete outcomes. Write N if it does not have discrete outcomes.

2. Toss a coin once.
   - D

3. Weigh a package.
   - N

4. Count the number of pages in a newspaper.
   - D

5. Count the number of miles a person jogs.
   - N

6. Pick a number from 1 to 1000.
   - D

7. Find the grams of sugar in an apple.
   - N
Key Concept  Normal Distribution

In a normal distribution,

- 68% of data fall within one standard deviation.
- 95% of data fall within two standard deviations.
- 99.7% of data fall within three standard deviations.

A normal distribution has a symmetric bell shape centered on the mean.

8. If babies’ weights are normally distributed around a mean weight, \(34\%\) of the weights should fall within one standard deviation above the mean.

Got It?  Zoology  The bar graph gives the weights of a population of female brown bears. The red curve shows how the weights are normally distributed about the mean, 115 kg. Approximately what percent of female brown bears weigh less than 120 kg?

9. Circle all intervals on the graph above that are less than 120 kg.

10. Estimate the percent for each interval.

Sample estimates: \(\leq 77: 0\%\); 80–89: 1\%; 90–99: 5\%; 100–109: 23\%; 110–119: 42\%

11. Add the percents.

\[1 + 5 + 23 + 42 = 71\]

12. About \(71\%\) of the female brown bears weigh less than 120 kg.
Problem 2  Sketching a Normal Curve

Got It? Zoology  For a population of female European eels, the mean body length is 21.1 in. The standard deviation is 4.7 in. Sketch a normal curve showing eel lengths at one, two, and three standard deviations from the mean.

Use the information from the problem to find each length.

13. one standard deviation from the mean
   \[
   \text{mean} - 1 \text{ deviation} = 21.1 - 4.7 = 16.4 \
   \text{mean} + 1 \text{ deviation} = 21.1 + 4.7 = 25.8
   \]

14. two standard deviations from the mean
   \[
   \text{mean} - 2 \text{ deviations} = 21.1 - 2 \times 4.7 = 11.7 \
   \text{mean} + 2 \text{ deviations} = 21.1 + 2 \times 4.7 = 30.5
   \]

15. three standard deviations from the mean
   \[
   \text{mean} - 3 \text{ deviations} = 21.1 - 3 \times 4.7 = 7 \
   \text{mean} + 3 \text{ deviations} = 21.1 + 3 \times 4.7 = 35.2
   \]

16. Sketch a normal curve showing the eel lengths at one, two, and three standard deviations.

Got It? Zoology  For a population of female European eels, the mean body length is 21.1 in. The standard deviation is 4.7 in. Sketch a normal curve showing eel lengths at one, two, and three standard deviations from the mean.

Use the information from the problem to find each length.

13. one standard deviation from the mean
   \[
   \text{mean} - 1 \text{ deviation} = 21.1 - 4.7 = 16.4 \
   \text{mean} + 1 \text{ deviation} = 21.1 + 4.7 = 25.8
   \]

14. two standard deviations from the mean
   \[
   \text{mean} - 2 \text{ deviations} = 21.1 - 2 \times 4.7 = 11.7 \
   \text{mean} + 2 \text{ deviations} = 21.1 + 2 \times 4.7 = 30.5
   \]

15. three standard deviations from the mean
   \[
   \text{mean} - 3 \text{ deviations} = 21.1 - 3 \times 4.7 = 7 \
   \text{mean} + 3 \text{ deviations} = 21.1 + 3 \times 4.7 = 35.2
   \]

16. Sketch a normal curve showing the eel lengths at one, two, and three standard deviations.

Problem 3  Analyzing a Normal Distribution

Got It? The scores on the Algebra 2 final are approximately normally distributed with a mean of 150 and a standard deviation of 15. What percentage of the students who took the test scored above 180?

Use the normal curve at the right for Exercises 17–19.

17. Use the information from the problem to label the mean and the standard deviations.

18. Label each standard deviation section with its percentage.

19. Shade the portion of the graph that corresponds to the percentage of students with scores above 180.
Lesson Check • Do you UNDERSTAND?

Reasoning  What is the effect on a normal distribution if the mean increases by 10? If the standard deviation increases by 10?

Use the following statement for Exercises 21 and 22. Write T for true or F for false.

A population is normally distributed with a mean of 34 and a standard deviation of 6.

21. Suppose the mean increases by 10.
   - The new mean is 44.  \( T \)
   - The new standard deviation is 16.  \( F \)
   - Each interval shifts 10 units to the right.  \( T \)

22. Suppose the standard deviation increases by 10.
   - The new mean is 44.  \( F \)
   - The new standard deviation is 16.  \( T \)
   - The width of each interval increases to 16.  \( T \)

Underline the correct word or number to complete each sentence about a normal distribution.

23. If the mean increases by 10, the bell curve shifts 10 units to the right / left.

24. If the mean increases by 10, each standard deviation shifts 10 units to the right / left.

25. If the standard deviation increases by 10, each interval becomes narrower / wider by 10 units.

Math Success

Check off the vocabulary words that you understand.

- [ ] discrete
- [ ] continuous
- [ ] normal distribution

Rate how well you can solve problems involving the normal distribution.

Need to review 0 2 4 6 8 10 Now I get it!